

Table 1 Eigenfrequencies and amplitudes of a 100-m-diam hoop-column antenna ($a = 100$ m, $e = 1$ m, $\alpha = 1$, $\omega = 1$)

| n | λ_n | A_n | λ_n (predicted by the simple model ³) |
|-----|-------------|------------------|---|
| 1 | 0.244 | $0.3200 \Pi^2 A$ | 0.119 |
| 2 | 0.398 | $0.4600 \Pi^2 A$ | 0.214 |
| 3 | 0.513 | $0.0016 \Pi^2 A$ | 0.271 |
| 4 | 0.603 | $0.0020 \Pi^2 A$ | 0.506 |
| 5 | 0.681 | — | 0.721 |
| 6 | 0.754 | — | 0.890 |
| 7 | 0.823 | — | 0.919 |
| 8 | 0.956 | — | 1.009 |

Table 2 Comparison between the simple model and the finite element model

| Antenna model | Ratio of eigenfrequencies | | | | | | |
|-----------------------------------|---------------------------|------|------|------|------|------|------|
| Simple model | 1.7 | 1.29 | 1.18 | 1.13 | 1.11 | 1.09 | 1.16 |
| Finite element model ³ | 1.8 | 1.26 | 1.87 | 1.42 | 1.13 | 1.03 | 1.18 |

$$+ \sum_{n=1}^{\infty} A_n \left[\frac{-Y_0(\lambda_n e) J_0(\lambda_n r)}{J_0(\omega e / \alpha) Y_0(\omega r / \alpha)} + Y_0(\lambda_n r) \right] \sin(\alpha \lambda_n t) \quad (3b)$$

The coefficients A_n , $n = 1, 2, \dots$, are to be obtained from the initial conditions (1b-1e) and the Bessel-Fourier series expansion and are given by⁴

$$A_n = \frac{\Pi^2 \lambda_n J_0(\lambda_n e) [L_1 - L_2 - L_3 + L_4]}{2\alpha [J_0^2(\lambda_n e) / J_0^2(\lambda_n a) - 1]} \quad (4a)$$

where

$$L_1 = \int_e^a r \bar{k}_1 Y_0\left(\frac{\omega r}{\alpha}\right) Y_0(\lambda_n e) J_0(\lambda_n r) dr \quad (4b)$$

$$L_2 = \int_e^a r \bar{k}_1 Y_0\left(\frac{\omega r}{\alpha}\right) J_0(\lambda_n e) Y_0(\lambda_n r) dr \quad (4c)$$

$$L_3 = \int_e^a r \bar{k}_2 J_0\left(\frac{\omega r}{\alpha}\right) Y_0(\lambda_n e) J_0(\lambda_n r) dr \quad (4d)$$

$$L_4 = \int_e^a r \bar{k}_2 J_0\left(\frac{\omega r}{\alpha}\right) J_0(\lambda_n e) Y_0(\lambda_n r) dr \quad (4e)$$

$$\bar{k}_1 = \omega A J_0(\omega e / \alpha) / b_0 \quad (4f)$$

$$\bar{k}_2 = \omega A Y_0(\omega e / \alpha) / b_0 \quad (4g)$$

$$b_0 = J_0(\omega e / \alpha) Y_0(\omega a / \alpha) - Y_0(\omega e / \alpha) J_0(\omega a / \alpha) \quad (4h)$$

provided $\lambda_n \neq \omega / \alpha$. When $\lambda_n = \omega / \alpha = \beta$, we obtain⁴

$$A_\beta = \frac{\Pi^2 J_0(\beta e) [L_{1\beta} - L_{2\beta} - L_{3\beta} + L_{4\beta}]}{2\alpha [J_0^2(\beta e) / J_0^2(\beta a) - 1]} \quad (5a)$$

where

$$L_{1\beta} = \bar{k}_1 Y_0(\beta e) \int_e^a J_0(\beta r) Y_0(\beta r) r dr \quad (5b)$$

$$L_{2\beta} = \bar{k}_1 J_0(\beta e) \int_e^a Y_0(\beta r) Y_0(\beta r) r dr \quad (5c)$$

$$L_{3\beta} = \bar{k}_2 Y_0(\beta e) \int_e^a J_0(\beta r) J_0(\beta r) r dr \quad (5d)$$

$$L_{4\beta} = \bar{k}_2 Y_0(\beta e) \int_e^a Y_0(\beta r) J_0(\beta r) r dr \quad (5e)$$

From Eqs. (3) and (4), we compute the values of λ_n and A_n for various values of n in Table 1. We note that the amplitudes A_n drop off sharply at high frequencies. The ratios of the frequencies are taken to cancel the effects of the scale factors (see Table 2).

Conclusions

The eigenfrequencies generated by the simple model compare favorably with the frequencies of the complex finite element model. Moreover, this analysis produces the amplitudes of the vibrating modes. These eigenfrequencies and eigenmodes could be used to study the dynamics and control system design of the hoop-column antenna as a flexible body.

Acknowledgments

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References

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- ³"Versatile Hoop/Column Antenna Structural Dynamic Model," Harris Corp., Melbourne, FL, Technical Rept., 1981, p. 50.
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Errata

Finite Element Method for Optimal Guidance of an Advanced Launch Vehicle

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